Efficiency in Matching Markets

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Basic Motivation

Will decentralized bargaining in labor markets get the “right” people into the “right” jobs?

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Will decentralized bargaining in labor markets get the “right” people into the “right” jobs?

Frictions like search costs and asymmetric information can be an impediment, but are there bargaining frictions too?

Even taking a very standard bargaining model, theory is ambiguous—it depends on the solution concept.

Demands empirical evidence, but this is hard to come by.

- Unobserved heterogeneity can rationalize any match as efficient.
- Hard to separate sources of inefficiency.

A laboratory experiment permits much greater control.

Design Overview

Remove non-bargaining frictions but let agreements be reached sequentially

- Alternative matches provide then endogenous "outside options"
- Market composition evolves as agreements are reached
- And bargaining positions can change
- Theory suggests this might lead to mismatch
- Does it?

Outline

Bargaining Protocol

Extend the canonical Rubinstein bargaining model to markets.

Model outline:

- Workers and firms in a market.
- Matters who matches to whom—heterogeneous surpluses.
- Each period, a player is selected at random to propose.
- Chooses whom to propose to, and what offer to make.
- Pair exit if proposal is accepted, otherwise remain in the market.
- Move to next period.

Discounting—game ends with probability 1% after each round.

We study three markets.

Market 1

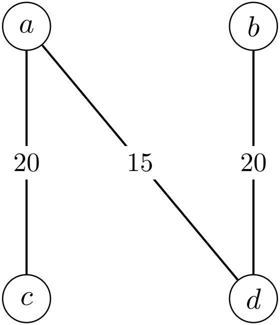


Figure: Market with 4 players, 2 workers (a, b) and 2 firms (c, d). Worker a can match to c, generating a surplus of 10, or to d generating a surplus of 15, while b can only match to d thereby generating a surplus of 10. It is efficient to a to match to c and for b to match to d.

Market 2

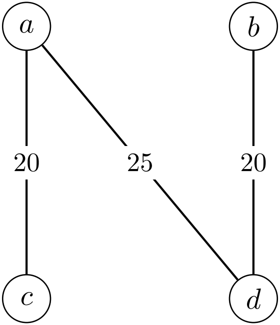


Figure: Market with 4 players, 2 workers (a, b) and 2 firms (c, d). Worker a can match to c, generating a surplus of 10, or to d generating a surplus of 25, while b can only match to d thereby generating a surplus of 10. It is efficient to a to match to c and for b to match to d.

Market 3

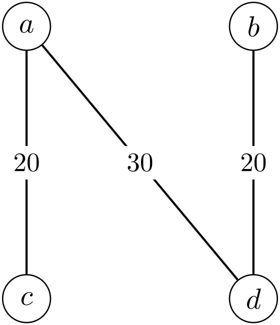


Figure: Market with 4 players, 2 workers (a, b) and 2 firms (c, d). Worker a can match to c, generating a surplus of 10, or to d generating a surplus of 30, while b can only match to d thereby generating a surplus of 10. It is efficient to a to match to c and for b to match to d.

Features of the design

Across all the markets we consider:

- There are only 4 players—minimal number required for bargaining positions to evolve.
- There is perfect information.
- Discounting, which can be interpreted as a friction, is very low.
- An efficient and perfectly equitable match is feasible.
- There is a corresponding extensive form game.
- Efficiency in a subgame perfect equilibrium requires increasingly complex strategies to be played.
- In the Markov Prefect Equilibria, inefficiency (rate of mismatch) increases across treatments.

Outline

Theory: Basic principles for finding the MPE

In an MPE if an offer is rejected, we stay in the same state.

And so continuation values don't change.

By perfection players make offers that are just acceptable.

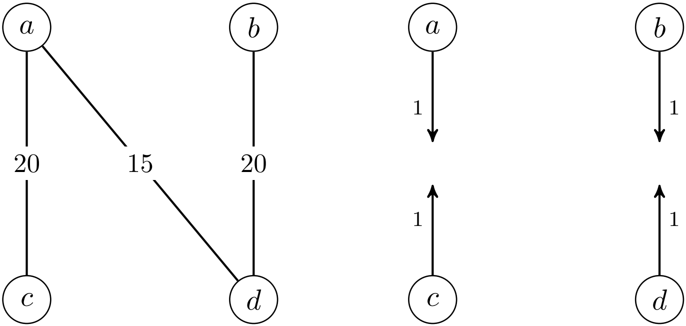
So when proposing a player can get the maximum payoff of:

1. their continuation value (by delaying or making an unacceptable offer);
2. the surplus from making a just acceptable offer to their efficient match; or
3. if they have a second match the surplus from making a just acceptable offer to their inefficient match.

An offer strategy profile pins down all continuation values.

Find a profile in which all offer strategies achieve their maximum.

Market 1



(a) Possible Matches. (b) Equilibrium

Figure: The unique Markov perfect equilibrium offer strategies are shown in Panel (b). Whomever is selected to propose makes a just acceptable offer to their efficient match with probability 1, and so there is no mismatch in equilibrium.

Solving for the MPE

Players a and d are in symmetric positions. We call them the strong players (subscript S).

Players b and c are in symmetric positions. We call them the weak players (subscript W).

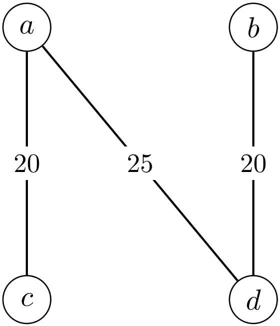
Given the offer strategies shown in the previous slide the continuation values of the players are:

$$\begin{aligned} V_S &= \frac{1}{4}\left(20 - \delta V_W\right) + \frac{3}{4}\delta V_S \\ V_W &= \frac{1}{4}\left(20 - \delta V_S\right) + \frac{3}{4}\delta V_W \end{aligned}$$

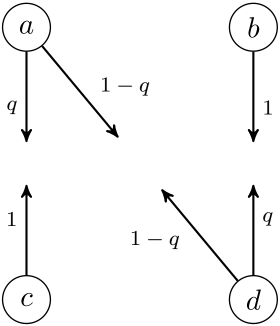
So, $\lim_{\delta \to 1} V_S(\delta) = \lim_{\delta \to 1} V_W(\delta) = 10$

Given these continuation payoffs, there is no profitable deviation.

Market 2



(a) Possible Matches.



(b) Equilibrium ($q \approx 1/2$)

Figure: The unique Markov perfect equilibrium offer strategies are shown in Panel (b). Players (b) and (c) make just acceptable offers to their efficient matches with probability 1, but players (a) and (d) mix between offering inefficiently to each other and making efficient offers.

Solving for the MPE

Let $W(\delta)$ be the continuation value of players in subgames where they are bargaining bilaterally with their efficient partners.

By Rubinstein (82) unique limit perfect equilibrium payoffs in these subgames are $\lim_{\delta \to 1} W(\delta) = 10$.

Let V_i be the continuation value of player i when no one has yet been matched.

Given offer strategies shown we have the following system of equations (including the indifference condition for mixing).

$$\begin{aligned}
V_S &= \frac{1}{4}\Big(20 - \delta V_W + \delta(1+q)V_S + (2-q)\delta W\Big) \\
V_W &= \frac{1}{4}\Big(20 - \delta V_S + (1-q)\delta V_W + (2-q)\delta W\Big) \\
20 - \delta V_W &= 25 - \delta V_S,
\end{aligned}$$

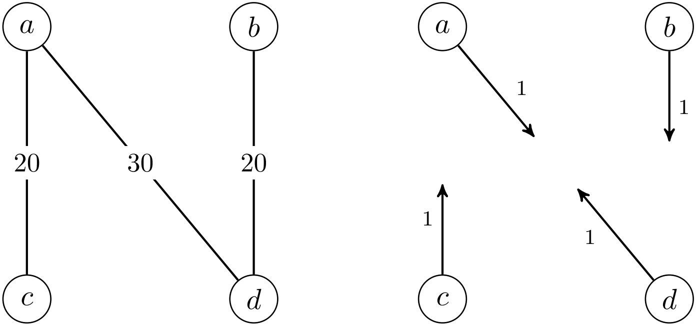
Solving for the MPE

Solving the above system:

- $\lim_{\delta \to 1} q(\delta) = \frac{16-\sqrt{160}}{6} = 0.56,$
- $\lim_{\delta \to 1} V_S(\delta) = 11.45,$
- $\lim_{\delta \to 1} V_W(\delta) = 6.45.$

Given these continuation payoffs, there is no profitable deviation.

Market 3



(a) Possible Matches. (b) Equilibrium

Figure: The unique Markov perfect equilibrium offer strategies are shown in Panel (b). Players (b) and (c) make just acceptable offers to their efficient matches with probability 1, but players (a) and (d) make just acceptable offers to each other leading to an inefficient match if one of them is selected to propose.

Solving for the MPE

Given offer strategies shown we have the following system of equations (including the indifference condition for mixing).

$$\begin{aligned} V_S &= \frac{1}{4}\Big(30 - \delta V_S + 2\delta V_S + \delta W\Big) \\ V_W &= \frac{1}{4}\Big(20 - \delta V_S + \delta W\Big) \end{aligned}$$

Solving the above system:

- $\lim_{\delta \to 1} V_S(\delta) = \frac{40}{3} = 13.33$,
- $\lim_{\delta \to 1} V_W(\delta) = \frac{25}{6} = 4.17$.

Given these continuation payoffs, there is no profitable deviation.

Source of inefficiency

Why don't the MPE offer strategies in Game 15 constitute an equilibrium in Game 25 or 30?

Suppose these strategies were played.

Then, one of the weak players would end up in a bilateral bargaining subgame and get 10.

A weak players could then guarantee 10 by deviating and matching second.

But then strong players would offer to each other.

If we start decreasing q, then V_W decreases.

In Game 25 V_W decreases until the strong players are indifferent.

In Game 30 there is a corner solution with $q = 0$.

Efficient Perfect Equilibria

Dropping the Markov restriction allows players to be punished and rewarded.

In Game 25, Markov reversion creates sufficient incentives to sustain efficient matches.

Strong players offer weak players their discounted MPE continuation value.

If they reject, play switches to the MPE and they are no better off.

This increases strong players' payoffs enough for them to not deviate and mismatch.

But it is not enough in Game 30.

Efficient Perfect Equilibria

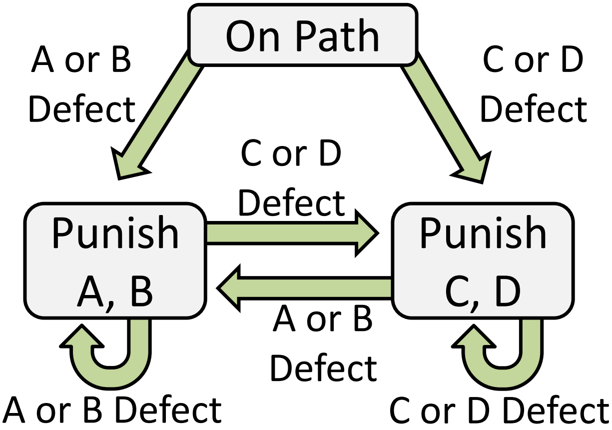
In Game 30 efficiency requires players to be punished and rewarded.

Suppose a strong player deviates by offering to the other strong player.

The receiver is rewarded for rejecting the offer, while the deviator is punished.

These rewards and punishments are supported by the threat to switch who is rewarded and punished.

Efficient Perfect Equilibria



Notes: The average efficiency levels and the corresponding 95% confidence intervals are reported for each game. Robust standard errors are obtained by clustering observations by session.

Some of the most relevant theoretical papers

Non-cooperative bargaining in finite matching markets

- Kranton and Minehart (00); Corominas Bosch (04); **Gale and Sabourian (06)**; Polanski (07); Okada (11); **Abreu and Manea (12a**,12b**)**; Polanski and Vega-Redondo (14); **Elliott and Nava (19)**.

Non-cooperative models in large matching markets

- Rubinstein and Wolinsky (85, 90), Gale (87), Binmore and Herrero (88), Gul (89), Moreno and Wooders (02); Manea (11,13); Lauermann (13).

Cooperative models of bargaining in matching markets

- Shapley and Shubik (71); Myerson (77); Crawford and Knoer (81); Rochford (84); Demange et al (86); Bennett (88); Elliott (14).

Outline

Experimental Set up

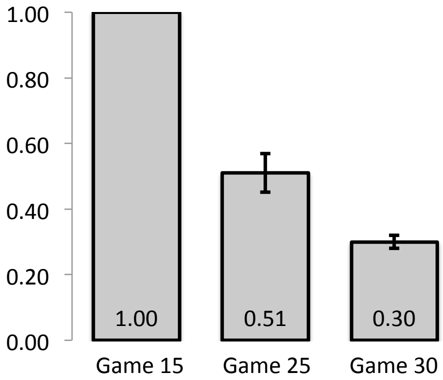
- Three treatments: Game 15 , Game 25, Game 30
- In each session one market game is played 10 times
- Focus on *experienced games* (last five times the game is played)
- Quiz about the rules of the game before experiment
- Total 176 subjects
  - Game 15: 3 sessions with 40 subjects total
  - Game 25: 4 sessions with 68 subjects total
  - Game 30: 3 sessions with 68 subjects total
- Groups and network places randomly assigned in each game
- Discounting: 1% chance game ends each round.

Experimental Design Choices

1. How to pay players
   - hedging motives across games if more than one game is paid
   - Azrieli, Chambers and Healy (Journal of Political Economy, 2018)
2. Strategy Method
   - eliciting offers from all market participants before selecting which one is implemented gives us 4 times more data
   - incentive compatible, b/c positive prob it will be implemented
   - for survey of the literature on strategy method see Brandts and Charness (Experimental Economics, 2011)
3. Implementing Discounting
   - absent discounting, hold-up problem might occur
   - Frechette and Yuksel (Experimental Economics, 2017) evaluate four ways to implemented discounting and random termination in the lab experiments

Outline

Efficiency (experienced games)



Notes: The average efficiency levels and the corresponding 95% confidence intervals are reported for each game. Robust standard errors are obtained by clustering observations by session.

Statistical analysis

$$\mathbb{1}_{\text{Eff}} = \beta_0 + \beta_1 \cdot \mathbb{1}_{\text{Game25}} + \beta_2 \cdot \mathbb{1}_{\text{Game30}} + \beta_3 \cdot \mathbb{1}_{\text{Game15}} \cdot \mathbb{1}_{\text{Strong First}} + \beta_4 \cdot \mathbb{1}_{\text{Game25}} \cdot \mathbb{1}_{\text{Strong First}} + \beta_5 \cdot \mathbb{1}_{\text{Game30}} \cdot \mathbb{1}_{\text{Strong First}} + \epsilon$$

Indicators:

- $\mathbb{1}_{\text{Eff}}$ *(match reached is efficient)*
- $\mathbb{1}_{\text{Game25}}$ *(Game 25 treatment)*
- $\mathbb{1}_{\text{Game30}}$ *(Game 30 treatment)*
- $\mathbb{1}_{\text{Strong First}}$ *(proposer is a strong player)*

Note: The coefficients (including joint hypothesis testing) of this regression answer many questions.

Efficiency (experienced games)

	Regression (1)	Regression (2)
Dependent Variable	Efficiency	Efficiency
Constant (β_0)	1.00^{***} (0.00)	1.00^{***} (0.00)
Game 25 (β_1)	-0.49^{***} (0.03)	-0.34*** (0.03)
Game 30 (β_2)	-0.70^{***} (0.01)	-0.47*** (0.04)
Strong First $\times$ Game 15 (β_3)		0.00 (1.00)
Strong First $\times$ Game 25 (β_4)		-0.37** (0.09)
Strong First $\times$ Game 30 (β_5)		-0.49*** (0.04)
# of obs	n=197	n=197
# of clusters	10	10
R-squared	0.2841	0.4238

Notes: Linear regressions with standard errors clustered at the session level are reported. The significance is indicated by *** and ** for 1% and 5% significance level.

Hypothesis tests for efficiency (experienced games)

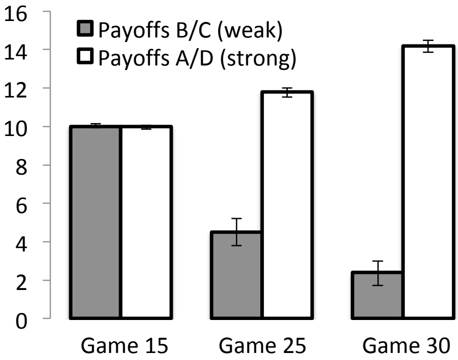
	Regression	Null Hypothesis	Alternative Hypothesis	P-Value
Test 1	Regression (1)	$\beta_0 + \beta_1 = \beta_0 + \beta_2$	$\beta_0 + \beta_1 > \beta_0 + \beta_2$	$p < 0.0001$
Test 2	Regression (2)	$\beta_4 = \beta_5$	$\beta_4 > \beta_5$	$p = 0.1042$
Test 3	Regression (1)	$\beta_0 + \beta_1 = 0.72$	$\beta_0 + \beta_1 < 0.72$	$p < 0.0001$
Test 4	Regression (1)	$\beta_0 + \beta_2 = 0.50$	$\beta_0 + \beta_2 < 0.50$	$p < 0.0001$

Interpretation

1. Positive values of β_1 and β_2—efficiency is lower in Games 25 and 30 than Game 15.
2. Test 1—efficiency declines from Game 25 to Game 30.
3. Negative values of β_4 and β_5 in Regression (2)—efficiency is lower when first proposer is strong in Games 25 and 30.
4. Test 2—first mover efficiency loss larger in Game 30.
5. Test 3—In Game 25 efficiency is lower than 72% (lower than predicted by the MPE)
6. Test 4—In Game 30 efficiency is lower than 50% (lower than predicted by the MPE)

- Qualitative results (1-4) consistent with MPE and inconsistent with efficient PE.
- Quantitative results (5-6) inconsistent with MPE.

Payoffs by network positions (experienced games)



Notes: The average payoffs and the corresponding 95% confidence intervals are reported for each game. Robust standard errors are obtained by clustering observations by session.

Statistical Analysis

$$\begin{aligned}\text{Payoff} &= \beta_0 + \beta_1 \cdot \mathbb{1}_{\text{Game25}} + \beta_2 \cdot \mathbb{1}_{\text{Game30}} + \beta_3 \cdot \mathbb{1}_{\text{Game15}} \cdot \mathbb{1}_{\text{Strong}} \\ &+ \beta_4 \cdot \mathbb{1}_{\text{Game25}} \cdot \mathbb{1}_{\text{Strong}} + \beta_5 \cdot \mathbb{1}_{\text{Game30}} \cdot \mathbb{1}_{\text{Strong}} \\ &+ \beta_6 \cdot \mathbb{1}_{\text{Game15}} \cdot \mathbb{1}_{\text{Exit First}} + \beta_7 \cdot \mathbb{1}_{\text{Game25}} \cdot \mathbb{1}_{\text{Exit First}} \\ &+ \beta_8 \cdot \mathbb{1}_{\text{Gaeme30}} \cdot \mathbb{1}_{\text{Exit First}} + \epsilon\end{aligned}$$

Indicators:

- $\mathbb{1}_{\text{Game25}}$ *(Game 25 treatment)*
- $\mathbb{1}_{\text{Game30}}$ *(Game 30 treatment)*
- $\mathbb{1}_{\text{Strong}}$ *(Player is strong)*
- $\mathbb{1}_{\text{Exit First}}$ *(Player exited the market first)*

Players' payoffs in experienced games

	Regression (3)	Regression (4)
Dependent Variable	Players' Payoffs (all players)	Players' Payoffs (strong players efficient matches)
Constant (β_0)	10.04^{***} (0.03)	9.97^{***} (0.02)
Game 25 (β_1)	-5.53^{***} (0.23)	0.13^{**} (0.05)
Game 30 (β_2)	-7.68^{***} (0.10)	-0.02 (0.04)
Strong × Game 15 (β_3)	-0.07 (0.05)	
Strong × Game 25 (β_4)	7.26^{***} (0.25)	
Strong × Game 30 (β_5)	11.81^{***} (0.14)	
Exit first × Game 15 (β_6)		-0.01 (0.02)
Exit first × Game 25 (β_7)		2.21^{***} (0.14)
Exit first × Game 30 (β_8)		4.62^{***} (0.23)
# of obs	n = 788	n=218
# of clusters	10	10
R-squared	0.6977	0.8067

Notes: Linear regressions with robust standard errors clustered at the session level. Regression (3) considers payoffs of all players, while Regression (4) focuses on the payoffs of strong player (those with two links, players A and D) in the markets that reached efficient outcome. Significance: *** and ** for 1% and 5% levels.

Hypothesis tests for players' payoffs (experienced games)

	Reg.	Test			Hypothesis Null	Hypothesis Alt.	P-Value
Test 5	(3)	$\beta_0 + \beta_1$	?	$\beta_0 + \beta_2$	$=$	$>$	$p < 0.0001$
Test 6	(3)	$\beta_0 + \beta_1 + \beta_4$	?	$\beta_0 + \beta_2 + \beta_5$	$=$	$<$	$p < 0.0001$
Test 7	(4)	β_7	?	β_8	$=$	$<$	$p < 0.0001$

Interpretation

1. β_3 insignificant—no evidence strong players do better in Game 15.
2. β_4 and β_5 positive—strong players receive higher payoffs than weak players in Game 25 and Game 30.
3. Test 5—weak players get higher payoffs in Game 25 than Game 30.
4. Test 6—strong players get higher payoffs in Game 30 than Game 25.
5. β_6—no evidence strong players do better moving first in Game 15.
6. β_7 and β_8 positive—strong players do better moving first in Games 25 and 30.
7. Test 7—strong players relative benefit of moving first is larger in Game 30 than in Game 25.

- Qualitative results (1-7) all consistent with MPE.

Predicted versus observed payoffs in experienced games

	Game 15		Game 25		Game 30	
	B (C)	A (D)	B (C)	A (D)	B (C)	A (D)
Theories						
MPE all	10	10	6.45	11.45	4.17	13.33
MPE $\mid$ eff.	10	10	8.95	11.05	8.34	11.67
Reversion	10	10	8.75	11.25	—	—
Carrot & Stick	10	10	$(7\frac{7}{9}, 9\frac{4}{9})$	$(10\frac{5}{9}, 12\frac{2}{9})$	$(6\frac{1}{9}, 9\frac{4}{9})$	$(10\frac{5}{9}, 13\frac{8}{9})$
Data						
all	10	10	4.5	11.8	2.4	14.2
	(0.03)	(0.03)	(0.25)	(0.10)	(0.11)	(0.05)
$\mid$ efficient	10	10	8.8	11.2	7.7	12.3
	(0.03)	(0.03)	(0.10)	(0.10)	(0.09)	(0.09)

Notes: The first two rows under the category of **Data** report players' payoffs and robust standard errors in all the final outcomes, while the last two rows under the category of **Data** focus on the groups that reached an efficient outcome.

Summary of results

- The MPE organizes the data well qualitatively, and does a better job than the efficient PE.
- The MPE does not do so well quantitatively.
- There is substantial mismatch—quantitatively this is *higher* than predicted by the MPE.

Outline

Are there systematic deviations from the MPE?

- If so, what are they and can an extended theory explain the data better?

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- If so, what are they and can an extended theory explain the data better?
- Studying strategies some weak players demand equitable splits. Incorporating this into the MPE improves the fit with the data.
- Can learn from the literature here on the ultimatum game—some people demand equality.
  - See Weg et al. (1990) and Kagel et al. (1996).

What drives the inefficiencies?

- What is driving the substantial inefficiencies?
  - Could it be the protocol—with more flexibility would subjects reach efficient outcomes?
  - Could it be the non-stationarity—with reneging the environment is more stationary, would this improve efficiency?

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  - Could it be the non-stationarity—with reneging the environment is more stationary, would this improve efficiency?
  - Run an experiment with reneging. Efficiency is improved, but some remains.

Cooperative Vs Non-cooperative Theory

Focus of this paper has been on the dynamics and how this impacts efficiency.

Have focused on non-cooperative theories.

Cooperative theories are typically static and efficient.

- e.g., Core, nucleolus, kernel, Pre-kernel, symmetrically pairwise bargained allocations (Rochford, 1984).

But interestingly sociology experiments tend to support core allocations.

- e.g., Cook et al. (1978), Cook et al. (1983), Bienenstock and Bonacich (1993), Skvoretz and Willer (1993).
- There is a nice albeit brief discussion in Jackson (2010).
- Focus on different questions, but setup is typically:
  - protocol free *with* reneging.