Beauty Contests and the Term Structure

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Motivation

Can information frictions help to explain the sizeable term premia contained in Treasury yields?

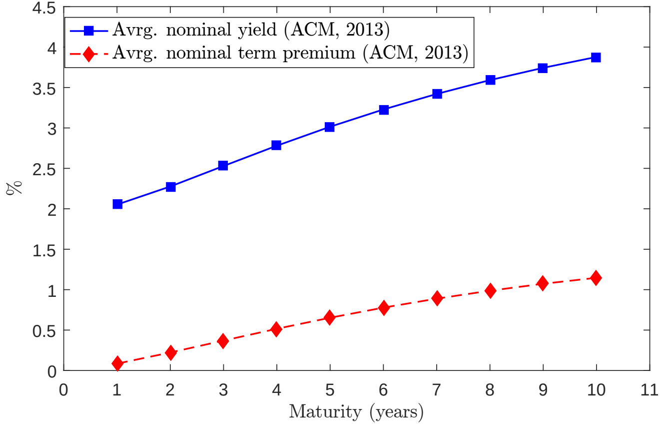

Figure: Zero-coupon US Treasury yield curve (4/1/1999 - 30/6/2017)

Literature

Bond premium puzzle

- Recursive preferences—Epstein and Zin (1989), Rudebusch and Swanson (2012), van Binsbergen et al. (2012)
- Model uncertainty—Barillas et al. (2009)
- Long-run risk—Bansal and Yaron (2004), Croce (2014)
- Rare disasters—Rietz (1988), Barro (2006)
- Habit Formation—Constantinides (1990), Campbell and Cochrane (1999), Rudebusch and Swanson (2008)
- Valuation Risk—Albuquerque et al. (2016)

Information in strategic settings and volatility

- Use of public information—Morris and Shin (2002), Angeletos and Pavan (2007)
- Volatility from information frictions—Angeletos and La'O (2013), Bergemann et al. (2015), Angeletos et al. (2018)

Overview

1. Decomposing the term premium
2. Models with a representative agent
3. Models with heterogeneously informed agents
4. A beauty contest model

Decomposing the term premium

Household side of generic DSGE model

- Representative household maximises

$$\mathrm{E}_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_s, l_s)$$

subject to

$$c_t + \sum_{n=1}^{N} p_t^{(n)} b_t^{(n)} = w_t l_t + d_t + \sum_{n=1}^{N} p_t^{(n-1)} b_{t-1}^{(n)}$$

- $b_t^{(n)}$—non-contingent default-free zero-coupon bonds with maturity $n = 1, 2, \ldots, N$
- $p_t^{(n)}$—bond price (note $p_t^{(0)} = 1$)

Decomposing the term premium

- Interior solution

$$p_t^{(n)} = \mathrm{E}_t m_{t+1} p_{t+1}^{(n-1)}, \qquad n \in \{1, 2, \ldots, N\}$$

  with stochastic discount factor (SDF) $m_{t+1} \equiv \beta \frac{u_c(c_{t+1}, l_{t+1})}{u_c(c_t, l_t)}$

- Implied yield

$$i_t^{(n)} = -\frac{1}{n} \ln p_t^{(n)}$$

  where we denote $i_t^{(1)} \equiv i_t$ for simplicity

- Hypothetical "risk-neutral price"

$$\tilde{p}_t^{(n)} = \mathrm{e}^{-i_t} \mathrm{E}_t \tilde{p}_{t+1}^{(n-1)}, \qquad n \in \{1, 2, \ldots, N\}$$

- Term premium (in per-period terms)

$$\psi_t^{(n)} = \frac{1}{n} \left(\tilde{p}_t^{(n)} - p_t^{(n)} \right)$$

Decomposing the term premium

Example – Two-period bond

- Term premium for $n = 2$

$$\psi_t^{(2)} = \frac{1}{2}\left(\tilde{p}_t^{(2)} - p_t^{(2)}\right) = -\frac{1}{2}\mathrm{Cov}_t\left(m_{t+1}, p_{t+1}^{(1)}\right)$$

- Take unconditional expectation and apply total covariance law to obtain following result

Proposition

Assume the law of iterated expectations holds and the stochastic discount factor m_{t+1} is in the household information set $\mathcal{I}_{t+1}$ at time $t+1$. The unconditional mean real term premium is given by

$$\mathrm{E}\psi_t^{(2)} = \frac{1}{2}\left[-\mathrm{Cov}\left(m_{t+1}, m_{t+2}\right) + \mathrm{Cov}\left(\mathrm{E}_t m_{t+1}, \mathrm{E}_{t+1} m_{t+2}\right)\right]$$

▸ $\psi_t^{(n)}$ ▸ $\mathrm{E}\psi_t^{(n)}$

Decomposing the term premium

Implications

- Mean term premium (for $n = 2$) can be decomposed into
  - covariance of successive *realisations* of the SDF
  - covariance of successive *expectations* of the SDF
- Result generalises to higher maturities ($n > 2$)
- Nominal term premium can be decomposed in analogous way
- So far theory focuses on first term (e.g. recursive preferences)
  $\Rightarrow$ Negative autocovariance of realisations of SDF required to explain positive mean term premium
- Process of expectation formation directly affects second term
  $\Rightarrow$ Positive autocovariance of expectations of SDF required to explain positive mean term premium

Next step

- Use decomposition to connect informational assumptions and term premia in analytical models

Models with a representative agent

Households, firms and technology

- Production function of representative firm

$$y_t = A_t \bar{l}^{1-\alpha}$$

- Technology $a_t \equiv \ln A_t$ follows

$$\begin{aligned} a_t &= x_t + \eta_t, \qquad && \eta_t \sim N(0, \sigma_\eta^2) \\ x_t &= \rho x_{t-1} + \varepsilon_t, && \varepsilon_t \sim N(0, \sigma_\varepsilon^2) \end{aligned}$$

- Representative household has logarithmic utility
  $\Rightarrow$ Coefficient of relative risk aversion tied to 1
- SDF can be expressed as

$$\begin{aligned} m_{t+1} = \beta \left(\frac{c_{t+1}}{c_t} \right)^{-1} &= \beta \left(\frac{A_{t+1} \bar{l}^{1-\alpha}}{A_t \bar{l}^{1-\alpha}} \right)^{-1} \\ &\approx \beta \left(1 + a_t - a_{t+1} \right) \end{aligned}$$

Models with a representative agent

Information sets

Model	$\subset \mathcal{I}_t$	$\not\subset \mathcal{I}_t$
Full information	m^t, a^t, x^t, η^t	
Partial information	m^t, a^t	x^t, η^t
Noisy information	m^t, s^t	a^t, x^t, η^t

Table: Information set of representative household

Notes: Signal given by $s_t = a_t + \xi_t$ with noise $\xi_t \sim N(0, \sigma_\xi^2)$.

Models with a representative agent

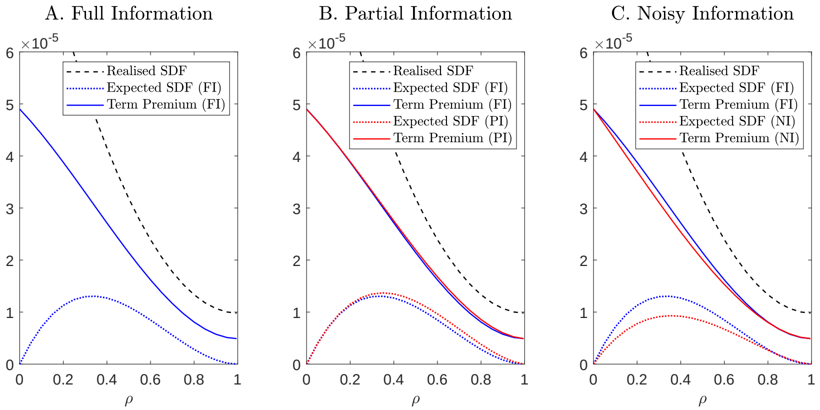

Figure: Components of mean real term premium ($n = 2$)

Notes: Solid line is mean real term premium, dashed line is component in autocovariance of realisations of SDF, dotted line is component in autocovariance of expected SDF. $\beta = 0.99$, $\mathrm{Var}(a_t) = 0.01^2$, $\mathrm{Var}(x_t)/\mathrm{Var}(a_t) = 0.9$, $\sigma_\xi^2 = \mathrm{Var}(a_t)/2$.

Models with heterogeneously informed agents

Identifying conditions required to generate term premia

- Heterogeneous information on the household-side now introduced to framework described before
- Continuum of ex ante identical agents indexed $i \in [0, 1]$
- Each agent observes signal $s_{i,t} = a_t + n_t + n_{i,t}$ and η_t allowing them to deduce

$$x_{i,t}^n = x_t + n_t + n_{i,t}$$

  but not x_t (persistent component of technology)
- Noise persistent so that

$$x_{i,t}^n = \rho x_{i,t-1}^n + \varepsilon_{i,t}^n$$

  where $\varepsilon_{i,t}^n \equiv \varepsilon_t + \xi_t + \zeta_{i,t} \sim N(0, \sigma_\varepsilon^2 + \sigma_\xi^2 + \sigma_\zeta^2)$
- Forming expectation about m_{t+1} requires inferring x_t from $x_{i,t}^n$

Models with heterogeneously informed agents

- Focus on symmetric linear equilibrium, in which expectations are formed according to

$$\hat{\mathrm{E}}_{i,t} x_t = \theta x_{i,t}^n \quad \forall i$$

- Term premium then given by

$$\psi_t^{(2)} = \frac{1}{2}\beta^2 \left[\theta(1-\rho)\sigma_\varepsilon^2 - \sigma_\eta^2\right]$$

$\Rightarrow \theta \uparrow$ implies $\psi_t^{(2)} \uparrow$

- Rational expectations are special case with $\theta = \theta^* = \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_\xi^2 + \sigma_\zeta^2}$
- Suppose $\hat{\mathrm{E}}_{i,t} x_t$ formed according to general loss function – Which conditions are required to obtain expectations consistent with the mean term premium in US data?

Models with heterogeneously informed agents

- General loss function

$$\mathrm{E}_{i,t}\left[\begin{pmatrix}\hat{\mathrm{E}}_{i,t}x_t & x_t & \int_0^1 \hat{\mathrm{E}}_{j,t}x_t dj\end{pmatrix}\begin{pmatrix}1 & \Omega_{12} & \Omega_{13}\\ 0 & \Omega_{22} & \Omega_{23}\\ 0 & 0 & \Omega_{33}\end{pmatrix}\begin{pmatrix}\hat{\mathrm{E}}_{i,t}x_t\\ x_t\\ \int_0^1 \hat{\mathrm{E}}_{j,t}x_t dj\end{pmatrix}\right]$$

- Optimal expectation satisfies

$$\hat{\mathrm{E}}_{i,t}x_t = \theta x_{i,t}^n = -\frac{1}{2}\left(\Omega_{12}\theta^* + \Omega_{13}\theta\frac{\sigma_\varepsilon^2+\sigma_\xi^2}{\sigma_\varepsilon^2+\sigma_\xi^2+\sigma_\zeta^2}\right)x_{i,t}^n$$

- Two degrees of freedom—If Ω_{12} is normalised to the value consistent with MSE minimisation (and hence RE),

$$\Omega_{13} = -2\left(\frac{\sigma_\varepsilon^2+\sigma_\xi^2+\sigma_\zeta^2}{\sigma_\varepsilon^2+\sigma_\xi^2}\right)\left(\frac{\theta-\theta^*}{\theta}\right)$$

$\Rightarrow \theta > \theta^*$ iff $\Omega_{13} < 0$

$\Rightarrow$ Sizeable term premium under strategic complementarity

A beauty contest model

Model

- More quantitative version of the model outlined just before
  - Labour supply endogenous (competitive labour market)

$$y_t = A_t L_t^{1-\alpha}$$

  - Household utility of more general form

$$u(c_{i,t}, l_{i,t}) = \frac{1}{1-\sigma}\left(c_{i,t} - \chi_0 \frac{l_{i,t}^{1+\chi}}{1+\chi}\right)^{1-\sigma}$$

  - Strategic complementarity through expectation formation in bond markets according to loss function with $\Omega_{12} = -2$ and $\omega = \Omega_{13}/(\Omega_{13} - 1)$, i.e.

$$(1-\omega)\mathrm{E}_{i,t}(\hat{\mathrm{E}}_{i,t}x_t - x_t)^2 - \omega \mathrm{E}_{i,t}\left(\int_0^1 \hat{\mathrm{E}}_{j,t}x_t dj\right)\hat{\mathrm{E}}_{i,t}x_t$$

  - Solution based on exact SDF rather than an approximation

A beauty contest model

Estimation approach

- US data, sample period 1999Q1-2017Q2
- Standard parameters calibrated (β, α, χ, χ_0)
- Remaining parameters estimated based on (simulated) method of moments
    - Parameters governing exogenous technology process (ρ,σ_η,σ_ε)
      $\Rightarrow$ Targets are the variance and first two autocovariances of detrended log consumption and variance of detrended log consumption growth
    - Parameters governing forecast formation and risk aversion (σ_ξ σ_ζ ω σ)
      $\Rightarrow$ Targets are the variance and autocovariance of the median forecast of productivity growth over the next ten years and term premium at one-year maturity

A beauty contest model

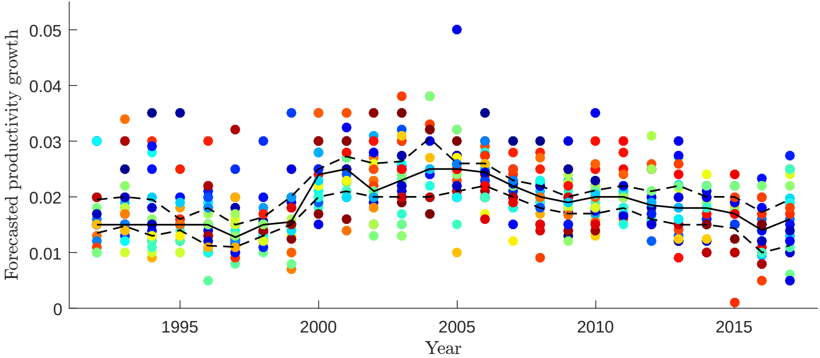

Figure: Forecasts of productivity growth from the SPF.

Notes: Solid line median, dashed lines lower and upper quartiles.

A beauty contest model

Parameter	Value	Description	Target (Data)
β	0.9997	Discount factor	$i^{(4)} = 0.0205 - 0.0191$ (Treasury yields, Adrian et al. (2013), 4/1/99 - 30/6/17; Inflation expectations, SPF, 1999q1-2017q2)
α	0.384	1 - Labour share	$1 - \alpha = 0.6160$ (Share of labour compensation in GDP, Penn World Table, 1999-2014)
χ	0.708	Inverse Frisch elasticity	$\mathrm{Var}(\ln l_t)/\mathrm{Var}(\ln c_t) = 0.3428$ (Consumption of nondurables and services, BEA; Population and hours, BLS, 1999q1-2017q2)
χ_0	2.04	Labour utility weight	$l = 1/3$

Table: Calibrated parameters

A beauty contest model

Parameter	Estimate	95% Confidence Interval	Description
ρ	0.90	$[0.81, 0.99]$	Shock persistence
σ_ε	2.0×10^{-3}	$[9.7 \times 10^{-4}, 3.1 \times 10^{-3}]$	SD innovation to persistent tech. component
σ_η	8.0×10^{-4}	$[0, 2.4 \times 10^{-3}]$	SD i.i.d. transitory tech. component
σ_ξ	9.9×10^{-5}	$[9.8 \times 10^{-5}, 1.0 \times 10^{-4}]$	SD innovation to common noise component
σ_ζ	2.2×10^{-3}	$[1.9 \times 10^{-3}, 2.5 \times 10^{-3}]$	SD innovation to idiosyncratic noise component
ω	0.80	$[0.78, 0.82]$	Strategic complementarity
σ	6.0	$[5.7, 6.3]$	Coefficient of relative risk aversion

Table: Estimated parameters

A beauty contest model

Moment	US data 1999Q1-2017Q2	Estimated model	Model with full information	Model with $\omega = 0$
Targeted				
$\mathrm{Var}\left(\hat{\gamma}_t^{50}\right)$	1.52×10^{-5}	1.42×10^{-5}	2.11×10^{-7}	4.68×10^{-8}
$\mathrm{Cov}\left(\hat{\gamma}_t^{50}, \hat{\gamma}_{t-4}^{50}\right)$	1.25×10^{-5}	9.29×10^{-6}	1.38×10^{-7}	3.06×10^{-8}
$\mathrm{E}\left(\hat{\gamma}_t^{75} - \hat{\gamma}_t^{25}\right)$	5.32×10^{-3}	5.40×10^{-3}	0	3.10×10^{-4}
$\mathrm{E}\psi_t^{(4)}$	8.2 bps	8.2 bps	2.6 bps	1.0 bps
Not targeted				
$\mathrm{E}\psi_t^{(8)}$	21.2 bps	16.0 bps	5.4 bps	2.0 bps
$\mathrm{E}\psi_t^{(12)}$	34.5 bps	21.1 bps	7.6 bps	2.7 bps
$\mathrm{E}\psi_t^{(16)}$	46.7 bps	24.4 bps	9.3 bps	3.3 bps
$\mathrm{E}\psi_t^{(20)}$	57.2 bps	26.7 bps	10.7 bps	3.7 bps

Table: Data and model moments

A beauty contest model

Estimation results

- Estimated beauty contest model
  - matches the moments related to consumption dynamics and volatility in hours almost perfectly
  - closely matches the moments targeted from the Survey of Professional Forecasters
  - delivers sizeable term premia, between 47 and 75 per cent of the nominal term premia in US data
- Model with full information (technology observed)
  - generates autocovariance in expectations that is two orders of magnitude too small
  - gives rise to term premia that are less than half of those in the beauty contest model
- Model without strategic complementarity ($\omega = 0$)
  - yields even lower autocovariance in expectations coinciding with even lower term premia

Conclusions

- The term premia contained in bonds of any maturity depend on autocovariance terms of the realisations and expectations of the stochastic discount factor
- Standard signal extraction problems in a representative agent framework generally do not give rise to sizeable term premia
- In a model with heterogeneously informed households and persistent noise, strategic complementarity in expectation formation can increase term premia
- An estimated model that allows for strategic complementarity is capable of explaining a substantial fraction of the term premia contained in the prices of US Treasuries

Decomposing the term premium

Proposition

Assume the law of iterated expectations holds and the stochastic discount factor m_{t+1} is in the household information set $\mathcal{I}_{t+1}$ at time $t+1$. The real term premium at maturity $n \in \{2, 3, \ldots\}$ is

$$\psi_t^{(n)} = \frac{1}{n} \sum_{k=0}^{n-2} \iota_t(k) \left[-\mathrm{Cov}_t \left(m_{t+k+1}, \prod_{j=k}^{n-2} m_{t+j+2} \right) + \mathrm{Cov}_t \left(\mathrm{E}_{t+k} m_{t+k+1}, \prod_{j=k}^{n-2} \mathrm{E}_{t+j+1} m_{t+j+2} \right) \right]$$

where

$$\iota_t(k) \equiv \begin{cases} 1 & \textit{for } k = 0 \\ \prod_{j=0}^{k-1} \mathrm{E}_t \mathrm{e}^{-i_{t+j}} & \textit{otherwise} \end{cases}$$

Decomposing the term premium

Lemma

Assume the law of iterated expectations holds and the stochastic discount factor m_{t+1} is in the household information set $\mathcal{I}_{t+1}$ at time $t+1$. The unconditional mean real term premium at maturity $n \in \{2, 3, \ldots\}$ is

$$
\begin{aligned}
&\mathrm{E}\psi^{(n)} = \\
&\frac{1}{n}\sum_{k=0}^{n-2}\Bigg\{\mathrm{E}\left(\iota_t(k)\right)\Bigg[-\mathrm{Cov}\left(m_{t+k+1}, \prod_{j=k}^{n-2} m_{t+j+2}\right) + \mathrm{Cov}\left(\mathrm{E}_t m_{t+k+1}, \mathrm{E}_t \prod_{j=k}^{n-2} m_{t+j+2}\right) + \\
&\mathrm{Cov}\left(\mathrm{E}_{t+k} m_{t+k+1}, \prod_{j=k}^{n-2} \mathrm{E}_{t+j+1} m_{t+j+2}\right) - \mathrm{Cov}\left(\mathrm{E}_t m_{t+k+1}, \mathrm{E}_t \prod_{j=k}^{n-2} \mathrm{E}_{t+j+1} m_{t+j+2}\right)\Bigg] + \\
&\mathrm{Cov}\left[\iota_t(k), -\mathrm{Cov}_t\left(m_{t+k+1}, \prod_{j=k}^{n-2} m_{t+j+2}\right) + \mathrm{Cov}_t\left(\mathrm{E}_{t+k} m_{t+k+1}, \prod_{j=k}^{n-2} \mathrm{E}_{t+j+1} m_{t+j+2}\right)\right]\Bigg\}
\end{aligned}
$$